

TESTING SUGRA UNIFIED MODELS *

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A brief review is given of the ways of testing SUGRA unified models and a class of string models using data from precision electroweak experiments, Yukawa unification constraints, and constraints from dark matter experiments. Models discussed in detail include mSUGRA, extended SUGRA model with non-universalities within $SO(10)$ grand unification, and effective theories with modular invariant soft breaking within a generic heterotic string framework. The implications of the Hyperbolic Branch including the focus point and inversion regions for the discovery of supersymmetry in collider experiments and for the detection of dark matter in the direct detection experiments are also discussed.

1. Introduction

It is a pleasure to dedicate this paper as a tribute to my thesis advisor
Pran Nath.

Over the past three decades supersymmetry¹ has come to play a central role in our exploration of physics beyond the standard model. However, a phenomenologically viable breaking of supersymmetry requires the framework of local supersymmetry which leads to supergravity. Considerable progress has taken place over the recent past in building models within (applied) supergravity and in string theory. In this paper we give here a brief analysis of testing supergravity unified models². We will also extend our discussion to low energy signatures of a class of models based on heterotic strings. First we will focus on supergravity (SUGRA) models, specifically the minimal supergravity (mSUGRA) and its extensions, which are currently among the leading candidates for physics beyond the standard model. A remarkable aspect of mSUGRA model is that the number

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of arbitrary parameters that appear in soft breaking in this model is much smaller³ than what is allowed in the minimal supersymmetric standard model (MSSM). Further, within mSUGRA one can achieve a unification of the gauge coupling constants⁴ and any small discrepancy between theory and experiment can also be resolved through gravitational corrections⁵. We begin by recalling briefly the basic ingredients that enter in the construction of supergravity unified models. The construction utilizes applied supergravity techniques^{2,6,7} coupling an arbitrary set of chiral superfields with a vector multiplet in the adjoint representation of the gauge group and then coupling the system with supergravity. As is well known the resulting Lagrangian depends on just three arbitrary functions: the gauge kinetic energy function $f_{\alpha\beta}$, the Kähler potential $K(z_i, z_i^\dagger)$ and the superpotential $W(z_i)$. Although there are a variety of scenarios such as gravity mediation, gauge mediation, anomaly mediation, breaking by anomalous $U(1)$'s etc, for the breaking of supersymmetry we focus here on the gravity mediated breaking. This utilizes a hidden sector and a visible sector, where supersymmetry is broken in the hidden sector and communicated to the visible sector by gravity^{2,8}. In the minimal version of the model based on a flat Kähler potential and a flat gauge kinetic energy function, the soft breaking sector of the theory is parametrized by the following parameters^{9,10}: the universal scalar mass m_0 , the universal gaugino mass $m_{1/2}$, the universal trilinear coupling A_0 , the bilinear coupling B_0 , and the Higgs mixing parameter μ_0 where μ_0 enters in the superpotential in the form $\mu_0 H_1 H_2$. Here H_2 gives mass to the up quark and H_1 gives mass to the down quark and the lepton.

One of the remarkable aspects of SUGRA model observed early on was the phenomenon that soft breaking can trigger breaking of the electroweak symmetry². This phenomenon appears quite naturally when the renormalization group effects are included as one moves from the grand unification scale to the electroweak scale^{11,12,13,14}. In this case the Higgs doublet mass-square, $m_{H_2}^2$ which turns negative triggers a breaking of the electroweak symmetry. In the analysis one takes into account also the one loop corrections to the effective potential^{15,16}. The minimization of the potential gives rise to two constraints arising from the conditions $\partial_{<H_1^0>} V = 0 = \partial_{<H_2^0>} V$. One of these constraints can be used to determine $|\mu_0|$ while the other allows one to eliminate B_0 in terms of $\tan \beta = < H_2 > / < H_1 >$. Thus after electroweak symmetry breaking mSUGRA is described by the parameters: m_0 , $m_{1/2}$, A_0 , $\tan \beta$, and $\text{sign}(\mu)$. In extended supergravity models one can include non-universalities in the Higgs sector and non-universalities of

the gaugino masses. Additionally there is also the possibility of extending SUGRA models to include CP phases. Large CP phases can indeed be compatible with the electric dipole moment (EDM) constraints¹⁷. In mSUGRA there are two such phases, but more phases appear in extended SUGRA models. These phases can affect a variety of low energy phenomenon such as analyses of dark matter. Another class of models studied widely are a variety of string based models, specifically models based on the heterotic string¹⁸. Of special interest to us here is the low energy effective theory that results from these models.

Such a low energy effective theory should take into account as much of the symmetry of the underlying string model as possible. One of these is the $SL(2, Z)$ large radius - small radius duality symmetry. Such a symmetry may be valid even non-perturbatively and thus it is reasonable to probe effective low energy supergravity theories which possess modular invariance. Investigation must of course include the stringent experimental constraints such as the flavor changing neutral current constraint given by the branching ratio $Br(b \rightarrow s + \gamma)$ ^{19,20,21,22}. The allowed parameter space of such models must also be consistent with the current constraints from the muon anomalous magnetic moment. Further, with R-parity invariance the lightest supersymmetric particle (LSP) in such models is absolutely stable. In SUGRA models and also in a class of string models the LSP in a significant part of the parameter space is the lightest neutralino, which is a strong candidate for cold dark matter (CDM).

The rest of the paper is organized as follows. In section Sec.2 we discuss the radiative electroweak symmetry breaking and discuss the hyperbolic branch of REWSB in SUGRA type of models in this context. In Sec.3 we discuss the precision data constraints. In Sec.4 we discuss the constraints of Yukawa unification on SUGRA models. In Sec.5 we discuss the constraints of dark matter and the possibility of detection of dark matter predicted by SUGRA and string models in direct detection experiments. Conclusions are given in Sec.6

2. Hyperbolic Branch from Radiative Electroweak Symmetry Breaking

In confronting models with high scale physics inputs like SUGRA and string models with experiments, one must evolve the SUGRA and string parameters including soft breaking parameters from a high scale down to the electroweak region. This evolution leads in a natural way breaking of the

$SU(2)_L \times U(1)_Y$ electroweak symmetry. This is the radiative electroweak symmetry breaking (REWSB) scenario. The resulting sparticle spectrum after REWSB carries the low energy signatures of the models. Thus it is important to examine the implications of REWSB in some detail. It turns out [23] that there are two regions of REWSB which are geometrically distinct and have widely different phenomenological implications. One of these is the ellipsoidal branch (EB) and the other is the hyperbolic branch (HB). In the following we discuss the origin of these branches.

In the analysis of REWSB one must take into account loop corrections in the effective potential as these corrections can become important in certain regions of the parameter space of models. Detailed numerical analyses, however, show that for the case $\tan\beta < 5$ the loop correction to the effective potential and well as the loop corrections to REWSB are small. Now typically the REWSB conditions arising from the minimization of the effective potential give a constraint which is quadratic in the soft parameters m_0 , $m_{1/2}$, A_0 and μ . Interestingly, for $\tan\beta < 5$ the REWSB constraint for fixed A_0 and fixed μ gives an ellipse in the $(m_0 - m_{1/2})$ plane. With μ having a lower limit, the implication of the constraint then is that for fixed A_0 and fixed μ there are upper limits on m_0 and $m_{1/2}$.

For $\tan\beta > 5$ the loop correction to the effective potential and to REWSB is not small and cannot be neglected. Fixing A_0 and μ one finds that in this case m_0 and $m_{1/2}$ lie on the boundary of a hyperbola. The qualitative difference between $\tan\beta < 5$ and $\tan\beta > 5$ cases can be understood as follows: For $\tan\beta > 5$, only $m_{H_2}^2$ rather than $m_{H_1}^2$ is important and the loop correction to the μ parameter, i.e., $\Delta\mu^2$ can be quite large at M_Z . It is seen that while μ^2 and $\Delta\mu^2$ each have large dependence on the scale at which the potential is minimized, their sum $\mu_{tot}^2 = \mu^2 + \Delta\mu^2$ remains reasonably flat with scale. It is then useful to carry out a minimization of the one-loop effective potential at a scale where $\Delta\mu^2$ becomes negligible relative to μ^2 (i.e. $\mu_{tot}^2 \sim \mu^2$). It turns out that the specific point where the loop correction turns out to be negligible lies essentially midway between the largest and the smallest sparticle mass of the model. Further, this scale is not far from the geometric mean of the stop masses, i.e., $\sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$ for most of the parameter space of models. A minimization of the effective potential at the scale where the one loop corrections are relatively small gives an insight into the REWSB constraint. Thus at the above scale one finds that either the co-efficient of m_0^2 or the co-efficient of $m_{1/2}^2$ in the quadratic constraints relating the soft parameters turns negative, thus changing an ellipse into a hyperbola for fixed μ and A_0 . Here, both $m_{1/2}$

and m_0 may extend to tens of TeV for μ fixed. The upper limit of m_0 for a given $m_{1/2}$ is then governed by the boundary defined by the lower limit of $m_{\tilde{\chi}_1^\pm}$ or REWSB. This boundary is quite sensitive to the top quark mass m_t . Another interesting insight is gotten if one interprets μ^2/M_Z^2 as a simple measure of fine-tuning. In this case one finds that even a small fine tuning can allow for large m_0 and $m_{1/2}$ on the hyperbolic branch.

The hyperbolic branch was further analyzed in the context of dark matter in Ref.24. It is found useful here to examine the composition of the LSP. One finds that the gaugino- Higgsino content of the LSP depends strongly on the region of the parameter space one is in. We consider the following distinct regions: (i) bino dominated scenario which typically arises in mSUGRA with m_0 and $m_{1/2}$ away from the low μ hyperbolic boundary. (ii) mixed gaugino-higgsino region which arises on the hyperbolic boundary with $m_{1/2} < 1.5$ TeV. This is the focus point region²⁵ where μ is small. (iii) higgsino dominated scenario in the inversion region of the hyperbolic boundary with $m_{1/2} > 2.5$ TeV or so. The hyperbolic region of very large m_0 and $m_{1/2}$ is explored in Fig.1. This resulted in the *inversion region* where the gaugino masses satisfy $m_i \gg |\mu|$, for $i = 1, 2, 3$. The masses of squarks, gluino, heavier neutralinos, heavier chargino and heavier Higgs bosons can be very large (even several tens of TeV) in this region and fall outside the reach of the Large Hadron Collider (LHC). The only light sparticles in the system aside from the light Higgs are the states $\tilde{\chi}_1^0$, $\tilde{\chi}_2^0$ and $\tilde{\chi}_1^\pm$ which are almost degenerate in mass. Because of this it turns out to be important to include lowest order perturbative corrections to these masses for a reliable prediction of phenomena involving these states.

Within the inversion region one finds $m_{\tilde{\chi}_1^0} < m_{\tilde{\chi}_1^\pm} < m_{\tilde{\chi}_2^0}$ at the tree level. While each of the above masses lie in the range of several hundred GeV to about 1 TeV, the mass differences $\Delta M^\pm = m_{\tilde{\chi}_1^\pm} - m_{\tilde{\chi}_1^0}$ and $\Delta M^0 = m_{\tilde{\chi}_2^0} - m_{\tilde{\chi}_1^0}$ are quite small ($O(10)$ GeV or smaller). As already pointed out, in the computation of these mass differences radiative corrections^{26,27} to the neutralinos and lighter chargino should also be included as they may have significant effects. However, the smallness of these mass differences poses a challenge regarding the observation of such particles. Although the observation of sparticles at colliders may be difficult on the inversion region of the hyperbolic branch, it may still be possible to observe dark matter in this region. This topic will be discussed in further detail in Sec.5.1.

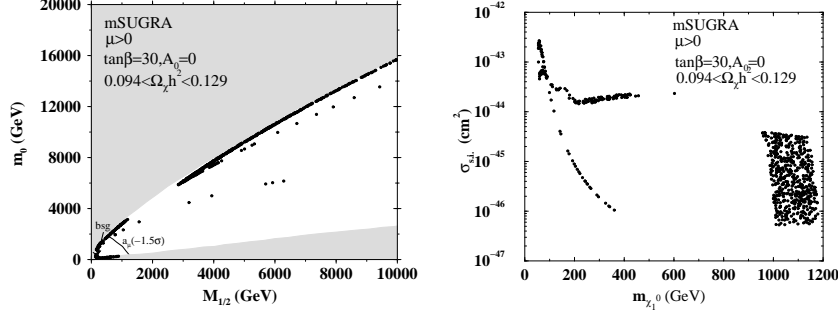


Figure 1. Dark matter in the multi-TeV Hyperbolic branch/Focus Point scenarios (From Ref.24). The WMAP satisfied relic density regions are shown in black. The WMAP satisfied smaller $m_{1/2}$ region (< 1.5 TeV) along the REWSB boundary (before the white region) falls in the focus point zone whereas the larger $m_{1/2}$ region (> 3 TeV) along the same boundary falls in the inversion zone of the hyperbolic branch.

3. Precision Data Constraints

In a number of processes the supersymmetric loop correction for physical observables may be comparable to the standard model loop correction. For such observables experiments that can probe the standard model correction can also be instrumental in testing corrections to models involving supersymmetry. An example of such an observable is the muon anomalous magnetic moment a_{μ} ^a. Here it was pointed out quite a while ago²⁸ that the supersymmetric electroweak contribution to the anomalous magnetic moment of the muon (a_{μ}^{SUSY}) may be as large or larger than the standard model electroweak contribution^b. Further, it was seen that the sign of a_{μ}^{SUSY} may be directly correlated with the sign of the μ parameter. Thus precision experiments designed to test the standard model electroweak correction would also test the supersymmetric correction, determine its sign and hence the sign of the μ parameter and also constrain the parameter space of mSUGRA, extended SUGRA and other competing models of soft breaking. The one loop supersymmetric correction to a_{μ} arises from the chargino and the neutralino exchanges. A detailed analysis of the supersymmetric correction within mSUGRA and other scenarios was carried out in Ref.30 (see also Refs.31,32) and the effect of CP phases was taken into account in later

^a a_{μ} is defined by the operator $\frac{e}{2m_{\mu}} a_{\mu} \bar{\mu} \sigma_{\alpha\beta} \mu F^{\alpha\beta}$.

^bIt turns out that the effect of extra compact dimensions on a_{μ} does not provide a big background to the supersymmetric correction and can be ignored in extracting the supersymmetric signal from data²⁹.

works³³. These analyses led to several important observations. Within the mSUGRA it was shown that the lighter chargino-sneutrino ($\tilde{\chi}_1^\pm - \tilde{\nu}_\mu$) loop is the dominating one relative to the heavier chargino-sneutrino ($\tilde{\chi}_2^\pm - \tilde{\nu}_\mu$) loop and all the neutralino-smuon ($\tilde{\chi}_i^0 - \tilde{\mu}$) loops. The dominance of the light chargino exchange results in a strong $\tan\beta$ dependence of a_μ^{SUSY} . It was further shown that $sign(a_\mu^{SUSY}) = sign(\mu)$ in mSUGRA^c. This particular relationship between the sign of a_μ^{SUSY} and the sign of μ is useful in the analysis of experimental data^{35,36d}. Additionally, in the theoretical predictions of a_μ^{SUSY} the $b \rightarrow s + \gamma$ constraint also plays an important role. The muon $g - 2$ data can also be useful to constrain other SUSY breaking scenarios like the minimal Anomaly Mediated Supersymmetry Breaking (mAMSB) model³⁸.

Over the past few years the experimental accuracy of $g_\mu - 2$ has increased very significantly. However, an extraction of new physics signal from the data requires a comparably accurate estimate of the hadronic correction which contributes to the standard model prediction. The computation of the hadronic correction has quite an interesting history and currently it is still the most ambiguous part of the the standard model prediction. The hadronic correction to the standard model prediction has many components. It consists of $O(\alpha^2)$ and $O(\alpha^3)$ hadronic vacuum polarization corrections, and corrections arising from the light by light contribution. The light by light correction has undergone a dramatic shift since it was shown that a switch of sign was needed^{39,40} over the previous evaluations. The other large source of error in the hadronic contribution comes from the $O(\alpha^2)$ vacuum polarization contribution. There are two independent recent numerical evaluations of this contribution. One of these is from Hagiwara *et. al.*⁴¹ and the other from Davier *et. al.*⁴². Hagiwara *et. al.* used the low energy data of $e^+e^- \rightarrow hadrons$ to compute the hadronic vacuum polarization contribution to a_μ^{SM} while Davier *et.al.* used τ decay data to compute the same. Using these two estimates of the vacuum polarization contributions, and including all other standard model contributions, one finds two different estimates for the difference between experiment (see Ref. (43)) and the standard model contribution. Assuming that the entire difference between experiment and the standard model arises from the supersymmetric contribution one finds the following two

^cThe generalized result is $sign(a_\mu^{SUSY}) = sign(\tilde{m}_2\mu)$. We follow the usual sign conventions for μ and A_0 as in Ref.34.

^dSee Ref.37 for additional works.

estimates for a_μ^{SUSY} : (i) $a_\mu^{SUSY} \equiv \Delta a_\mu(e^+e^-) = (23.9 \pm 10.0) \times 10^{-10}$, and (ii) $a_\mu^{SUSY} \equiv \Delta a_\mu(\tau \text{ decay}) = (7.6 \pm 9.0) \times 10^{-10}$. Clearly, a 2.4σ deviation from the SM result arises using the analysis of Hagiwara *et. al.*, whereas there exists no discrepancy between experiment and the standard model result for the evaluation of Davier *et. al.*. Here we adopt the result of case (i) which uses the e^+e^- data. In Fig.(2) an updated 1σ contours of a_μ^{SUSY} is shown using the constraint of case (i). The conclusion of this analysis is quite similar to what was obtained in Ref.36.

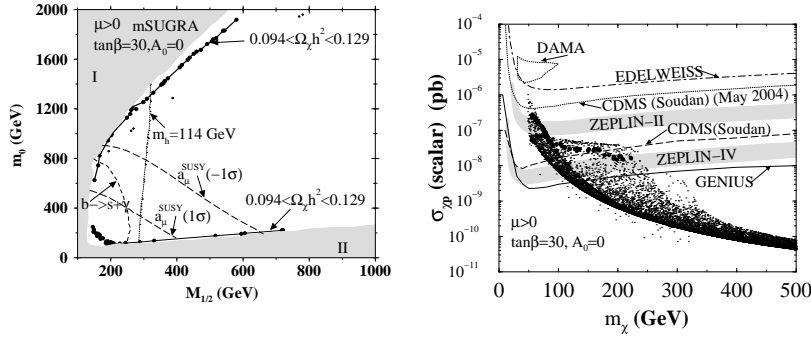


Figure 2. Present constraints on mSUGRA $m_{1/2} - m_0$ plane and spin-independent $\tilde{\chi}_1^0 - p$ cross section vs $m_{\tilde{\chi}_1^0}$ for $\tan\beta = 30$. The black regions lines/circles are the valid relic density region from WMAP. Contours are shown for the limits from a_μ^{SUSY} (via $\Delta a_\mu(e^+e^-)$) and from $Br(b \rightarrow s + \gamma)$ (left region disallowed) besides the LEP Higgs bound of ~ 114 GeV⁴⁴. The present and future dark matter detection limits are also shown. The top gray region in the $m_{1/2} - m_0$ plane is discarded by the REWSB constraint and the bottom gray zone is the stau-LSP region.

4. $h_b - h_\tau$ Yukawa Unification, a_μ^{SUSY} , $b \rightarrow s + \gamma$ and Nonuniversality

Typically $h_b - h_\tau$ Yukawa unification requires a negative μ ^{45,46}. This is so because $h_b - h_\tau$ unification demands $m_b(M_Z)$ in a range where there is a negative SUSY loop correction Δ_b to the b-quark mass⁴⁷.

$$m_b(M_Z) = \lambda_b(M_Z) \frac{v}{\sqrt{2}} \cos\beta(1 + \Delta_b) \quad (1)$$

The dominant supersymmetric contribution to Δ_b comes from the gluino and chargino exchanges so that

$$\Delta_b \simeq \Delta_b^{\tilde{g}} + \Delta_b^{\tilde{\chi}^\pm}$$

where $\Delta_b^{\tilde{g}} \propto \tan \beta \mu M_{\tilde{g}}$ and $\Delta_b^{\tilde{\chi}^{\pm}} \propto \tan \beta \mu A_t$. Each of these contributions are proportional to μ and it turns out that a negative Δ_b arises for negative μ . Thus $\mu < 0$ is preferred in mSUGRA for $h_b - h_\tau$ unification. This is somewhat at odds with the BNL $g - 2$ data which appears to indicate a $\mu > 0$ (see, however, Refs.48, 49, 50). However, it is fairly easy to reconcile these two results if we allow for non-universality of the gaugino masses. This comes about as follows: we can choose \tilde{m}_2 to be negative and $m_{\tilde{g}}$ to be positive. This leads to a positive a_μ^{SUSY} while Δ_b is negative. A solution of this type was analyzed in Ref.51. Such a phenomenon arises naturally in specific groups representations in $SU(5)$ and $SO(10)$ to which the gauginos may belong[51].

Non-universality of gaugino masses may arise from a non-trivial gauge kinetic energy function^{52,53,54} $f_{\alpha\beta}$ where $f_{\alpha\beta}$ transform according to a symmetric product of the adjoint representation of the gauge group. Thus in general $f_{\alpha\beta}$ has a non-trivial field content. For $SU(5)$, one gets the symmetric product as the sum of the following representations:

$$(24 \times 24)_{sym} = 1 + 24 + 75 + 200 \quad (2)$$

In general one has $\tilde{m}_i(M_G) = m_{1/2} \sum_r C_r n_i^r$, where n_i^r for $i = 1, 2, 3$ is the characteristic of the representation r and C_r is the relative weight. As an example, let us suppose that one retains only the 24 plet representation on the right hand side of Eq.(2). In this case one gets the following ratio of gaugino masses at the GUT scale: $\tilde{m}_3(M_G) : \tilde{m}_2(M_G) : \tilde{m}_1(M_G) = 2 : -3 : -1$. We see now that \tilde{m}_2 and \tilde{m}_3 have opposite signs. Thus if the gauginos transform as the 24 plet of $SU(5)$ then one can have both $h_b - h_\tau$ unification as well as a positive a_μ^{SUSY} for either sign of μ . However, the FCNC constraint from $b \rightarrow s + \gamma$ eliminates more parameter space for $\mu < 0$ than for $\mu > 0$. Including both signs of μ , $h_b - h_\tau$ unification then occurs for the following range of parameters: $\tan \beta = 15 - 45$ for $\delta_{b\tau} < 0.3$, where $\delta_{b\tau} = (|\lambda_b - \lambda_\tau|)/\lambda_\tau$. For $\delta_{b\tau} < 0.05$, one has $\tan \beta \sim 30$.

Next let us consider the $SO(10)$ case. One has the following result for the symmetric product of the adjoint representations:

$$(45 \times 45)_{sym} = 1 + 54 + 210 + 770 \quad (3)$$

There are many ways in which the $SO(10)$ gauge group can break down to the Standard Model gauge group. We consider the following two patterns

$$SO(10) \rightarrow SU(4) \times SU(2) \times SU(2) \rightarrow SU(3) \times SU(2) \times U(1) \quad (4)$$

10

and

$$SO(10) \rightarrow SU(2) \times SO(7) \rightarrow SU(3) \times SU(2) \times U(1) \quad (5)$$

As an illustration suppose that the symmetric product transforms like the 54 plet on the right hand side of Eq.(3). In this case Eq.(4) gives the following ratios for the gaugino masses at the GUT scale⁵⁵

$$\tilde{m}_3(M_G) : \tilde{m}_2(M_G) : \tilde{m}_1(M_G) = 1 : -3/2 : -1 \quad (6)$$

Next suppose we consider that the pattern of case Eq.(5). In this case we have the following ratio of gaugino masses

$$\tilde{m}_3(M_G) : \tilde{m}_2(M_G) : \tilde{m}_1(M_G) = 1 : -7/3 : 1 \quad (7)$$

We note that in each of the two cases above \tilde{m}_2 and \tilde{m}_3 have opposite signs. Thus it is possible to reconcile $h_b - h_\tau$ unification, $a_\mu^{SUSY} > 0$ and the $b \rightarrow s + \gamma$ constraint for these. An analysis of SUSY dark matter constraints and detection prospects for dark matter for these cases are analyzed in Ref.56. A further discussion of this topic will be given in Sec. 5.2.

5. Relic Density and Detection Rates

The current astrophysical data strongly indicates the existence of cold dark matter in the universe⁵⁷. The Wilkinson Microwave Anisotropy Probe (WMAP) experiment measured the parameters of the standard cosmological model⁵⁸. These are: $\Omega_b = 0.044 \pm 0.004$, $\Omega_m = 0.27 \pm 0.04$ and $\Omega_\Lambda = 0.73 \pm 0.04$. Here, $\Omega_{b,m} = \rho_{b,m}/\rho_c$ where $\rho_{b,m}$ is the baryon (matter) density and $\rho_c (= 3H_0^2/(8\pi G_N))$ is the critical mass density required to close the universe. Here H_0 is the Hubble parameter and $h = H_0/(100 \text{ km/s/Mpc})$ amounts to $h = 0.71^{+0.04}_{-0.03}$. Ω_Λ comes from the dark energy contribution. The cold dark matter density is then given by $\Omega_{CDM}h^2 = 0.1126^{+0.008}_{-0.009}$, which at 2σ level gives the following limit.

$$0.094 < \Omega_{CDM}h^2 < 0.129 \quad (8)$$

A specially attractive CDM candidate in R-parity conserved scenario of supersymmetric models is the lightest neutralino ($\tilde{\chi}_1^0$)⁵⁹. In supergravity models $\tilde{\chi}_1^0$ becomes the LSP for most of the region of the parameter space (see Refs.60, 57 for recent reviews) and one may consider $\Omega_{CDM} \equiv \Omega_{\tilde{\chi}_1^0} = \rho_{\tilde{\chi}_1^0}/\rho_c$. We should note here that the upper side of $\Omega_{CDM}h^2$ is a

^e1 pc = 3.2615 light year = 3.0856×10^{18} cm

strong limit but the validity of the lower bound becomes weak if we accept other candidates of dark matter. At high temperature of the early universe ($T \gg m_{\tilde{\chi}_1^0}$), $\tilde{\chi}_1^0$ was in thermal equilibrium with its decay products. The $\tilde{\chi}_1^0 \tilde{\chi}_1^0$ decay products are fermion pairs ($f\bar{f}$), gauge boson pairs (W^+W^- & ZZ), Higgs boson pairs ($hh, HH, AA, hH, hA, HA, H^+H^-$) or gauge boson-Higgs boson pairs (Zh, ZH, ZA & $W^\pm W^\mp$) with decays occurring through s, t and u channel diagrams⁶¹.

In determining the relic density of neutralinos at the current time we follow the standard procedure. Thus as the universe cools, the annihilation rate falls below the expansion rate of the universe and $\tilde{\chi}_1^0$ moves away from thermal equilibrium (freeze-out). $\Omega_{\tilde{\chi}_1^0} h^2$ can thus be computed for its present value by solving the Boltzmann equation for $n_{\tilde{\chi}_1^0}$, the number density of the LSP in a Friedmann-Robertson-Walker universe. Computing neutralino relic density most importantly involves computing $\langle \sigma_{eff} v \rangle$, where σ_{eff} is the neutralino annihilation cross section (which involves many final states, hence computationally it becomes quite intricate) and v is the relative velocity between two neutralinos annihilating each other. If there are species with mass close to the LSP mass, then one must include coannihilation processes^{62,63,64,65,66,67}.

In MSSM the lightest neutralino, i.e., the LSP, is composed of the bino, the wino (which are superpartners of U(1) and the $SU(2)_L$ gauge bosons) and two Higgsinos⁶⁸.

$$\tilde{\chi}_1^0 = N_{11}\tilde{B} + N_{12}\tilde{W}_3 + N_{13}\tilde{H}_1^0 + N_{14}\tilde{H}_2^0 \quad (9)$$

Here the coefficients N_{ij} are elements of the matrix that diagonalizes the neutralino mass matrix. The gaugino fraction of $\tilde{\chi}_1^0$ is defined by $F_g = |N_{11}|^2 + |N_{12}|^2$. A $\tilde{\chi}_1^0$ is gaugino like if F_g is very close to 1 (>0.9), higgsino like if $F_g < 0.1$. Otherwise it is a gaugino-higgsino mixed state.

In mSUGRA the composition of the lightest neutralino in various regions of the parameter space is as follows: (i) bulk region: In the bulk region of the $(m_{1/2} - m_0)$ plane i.e. the region between the REWSB boundary (on the larger m_0 side) and the $\tilde{\tau}_1$ turning LSP boundary (on the smaller m_0 limit) $\tilde{\chi}_1^0$ is gaugino-like or more specifically bino-like. (ii) Focus Point region: In the focus point (FP) region of the hyperbolic branch the LSP is in general a gaugino-higgsino mixed state. (iii): Inversion region (IR): In the inversion region of the hyperbolic branch the neutralino is essentially in a purely higgsino state. Typically in the bulk parameter space of mSUGRA the relic density is usually too large to satisfy Eq.(8) except in the *bulk annihilation region* characterized by low m_0 and low $m_{1/2}$ values (see Fig.2).

Coannihilations reduce the relic density appreciably in specific regions of the parameter space so as to satisfy Eq.(8). Stau coannihilations⁶² may reduce the relic density appreciably to satisfy the WMAP data and the WMAP allowed region falls near the $\tilde{\tau}$ -LSP boundary region with smaller m_0 values. These coannihilations processes are of the type $\tilde{\chi}_1^0 \tilde{\ell}_R^a \rightarrow \ell^a \gamma, \ell^a Z, \ell^a h$, $\tilde{\ell}_R^a \tilde{\ell}_R^b \rightarrow \ell^a \ell^b$, and $\tilde{\ell}_R^a \tilde{\ell}_R^{b*} \rightarrow \ell^a \bar{\ell}^b, \gamma\gamma, \gamma Z, ZZ, W^+ W^-, hh$. Here $\tilde{\ell}$ is effectively a stau. The coannihilations for the HB/FP/IR regions are described in Sec.5.1. The neutralino-stop coannihilations however occur for very limited values of A_0 ⁶³. Relic density is also satisfied for large $\tan\beta(>45)$ cases when $m_A \sim 2m_{\tilde{\chi}_1^0}$ ⁶⁹ which is associated with a large annihilation of type $\tilde{\chi}_1^0 \tilde{\chi}_1^0 \rightarrow A \rightarrow f\bar{f}$.

5.1. Dark Matter in the Focus Point and Inversion Regions of the Hyperbolic Branch

A study of neutralino relic density with the WMAP data along with using other phenomenological constraints for the regions (i)-(iii) above was made in Ref.24 (other post-WMAP analyses may be seen in Ref.70). Computation of $\Omega_{\tilde{\chi}_1^0} h^2$ in the focus point region²⁵ and the inversion region²⁴ of the hyperbolic branch particularly shows the existence of strong coannihilations of the LSP with lighter chargino $\tilde{\chi}_1^\pm$. Some of the dominant coannihilation processes in these region are^{66,64}: $\tilde{\chi}_1^0 \tilde{\chi}_1^+, \tilde{\chi}_2^0 \tilde{\chi}_1^+ \rightarrow u_i \bar{d}_i, \bar{e}_i \nu_i, A W^+, Z W^+, W^+ h$; $\tilde{\chi}_1^+ \tilde{\chi}_1^-, \tilde{\chi}_1^0 \tilde{\chi}_2^0 \rightarrow u_i \bar{u}_i, d_i \bar{d}_i, W^+ W^-$. Having the smallest mass difference between coannihilating sparticles the process $\tilde{\chi}_1^0 \tilde{\chi}_1^+$ indeed dominates among the above channels. As a result $\Omega_{\tilde{\chi}_1^0} h^2$ is reduced appreciably so that it satisfies Eq. (8) or coannihilations may even reduce it further (below the lower limit of Eq. (8)) thus causing $\tilde{\chi}_1^0$ to be a sub-dominant component of dark matter. Such *sub-dominant* region falls between the FP and the inversion region near the REWSB boundary. The analysis of Ref.24 also included the constraint from $b \rightarrow s + \gamma$ and a_μ^{SUSY} . $B_s^0 \rightarrow \mu^+ \mu^-$ limit was also included for possible large $\tan\beta$ implications. The LSP mass limit satisfying the WMAP data which arise from the neutralino-stau coannihilation is seen to be as large as 500 GeV for all $\tan\beta$. The same limit when the inversion region of HB is considered goes to about 1200 GeV. However, we should keep in mind that even a moderate amount of supersymmetric contribution to $(g-2)_\mu$ can eliminate the entire inversion region of the hyperbolic branch.

Also of interest is the spin-independent ($\sigma_{\tilde{\chi}_1^0 p}^{SI}$) and spin-dependent ($\sigma_{\tilde{\chi}_1^0 p}^{SD}$) neutralino-proton scattering cross sections^{71,72,73,60,61}. Among these

the former has more current interest from experiments like CDMS, EDELWEISS, ZEPLIN and GENIUS⁷⁴. The results show that the FP region is quite accessible in future experiments, but the inversion region will not be probed effectively. Typically a heavily higgsino-dominated region can be probed via indirect detection of dark matter^{75,76} experiments like IceCubes and ANTARES⁷⁷. It is expected that a narrow band within the inversion region may be probed similarly for indirect detection.

5.2. Dark Matter and Yukawa unification

Yukawa unification imposes additional constraints and there exist only few works which have taken into account such a constraint in the analysis of dark matter^{78,56,79}. Specifically in Ref.56 an analysis was carried out with non-universal gaugino masses. Within the SU(5) scenario with non-universal gaugino masses only the representation r_{24} allows $h_b - h_\tau$ unification and satisfies a_μ^{SUSY} as well as $b \rightarrow s + \gamma$ constraints. The sparticle spectrum is quite light in this case. Thus for $\mu > 0$ and $\delta_{b\tau} < 0.3$ one finds $m_{\tilde{\chi}_1^0} < 65$ GeV, $\Omega_{\tilde{\chi}_1^0} h^2$ lies in the desired range, and $\sigma_{\tilde{\chi}_1^0 p}^{SI}$ also lies in a range accessible to future dark matter experiments. For the SO(10) case where the gaugino mass matrix transforms like the 54 plet representation of $(45 \times 45)_{\text{sym}}$, the analysis give results similar to that of the SU(5) case. In this case one finds that for $\mu > 0$ and $\delta_{b\tau}, \delta_{tb}, \delta_{t\tau} < 0.3$, one has $m_{\tilde{\chi}_1^0} < 80$ GeV and the analysis of neutralino relic density and the spin-independent cross-section $\sigma_{\tilde{\chi}_1^0 p}^{SI}$ are similar to the SU(5) case discussed above. Regarding the collider reaches, all the sparticles in above scenarios can be completely probed at the LHC.

5.3. Dark Matter with Modular Invariant Soft Breaking

We discuss now dark matter within the context of an effective low energy supergravity theory which has an $SL(2, Z)$ modular invariance associated with a large radius- small radius duality symmetry. For simplicity we will assume that the Kähler potential depends on the dilaton field S and the Kähler moduli fields T_i ($i=1,2,3$). It is then easily seen that soft breaking in such a model⁸⁰ arising from spontaneous breaking of supersymmetry such as the one that arises via a hidden sector in supergravity theories is also modular invariant. Further, quite remarkably one finds that the constraints of radiative electroweak symmetry breaking in such a model determines $\tan \beta$ [81, 80]. The phenomenon that $\tan \beta$ is no longer a free parameter but rather is a determined quantity under the constraints of REWSB should

apply to a broader class of models, for example, to soft breaking in models based on intersecting D branes⁸². An analysis of dark matter within this class of models was carried out and the neutralino relic density computed⁸¹. Quite remarkably it is found that the modular invariant theory with $\tan\beta$ no longer a free parameter can satisfy the WMAP constraints. Further, it is found that the case with $\mu > 0$ where the WMAP constraints are satisfied leads to a dilaton dominated region while the case with $\mu < 0$ where the WMAP constraints are satisfied leads to a moduli dominated region. The $b \rightarrow s + \gamma$ constraint further restricts the parameter space of the model. For the $\mu > 0$ case one finds that the WMAP and the $b \rightarrow s + \gamma$ constraints lead to upper bounds on the sparticle masses and quite remarkably these lie within the reach of the LHC. However, this does not hold for the $\mu < 0$ case where the limits are much higher and some may lie outside the reach of the LHC. It is also interesting to analyze the possibilities for the direct detection of dark matter predicted within modular invariant supergravity theory. Here an analysis of the spin-independent neutralino-proton cross section ($\sigma_{\tilde{\chi}_1^0 p}^{SI}$) shows that these cross sections can be successfully probed in the current and future dark matter detectors [81]. The analysis of Ref.81 is indeed the first work to use the dual constraints of modular invariance and radiative breaking of the electroweak symmetry for a determination of $\tan\beta$ and utilizes such a determination for the analysis of sparticle spectra and dark matter. [For other phenomenological analyses of soft-breaking using modular invariance see Ref.83].

6. Conclusion

In this paper we have discussed possible tests of SUGRA and string models using precision data and data from dark matter experiment. We also discussed the constraints of Yukawa unification. The analysis was done in a variety of supergravity based models, which include mSUGRA, extended SUGRA with gaugino mass non-universalities, and models with modular invariant soft breaking in generic heterotic string models. In comparing theory with experiment one must ascertain to a high degree of accuracy the predictions of the standard model so that one may reliably determine the deviation between experiment and the standard model result. The case in point is the BNL experiment which has recently produced a value of $(g_\mu - 2)$ with a significant improvement over the previous determinations. However, a determination of whether or not a new physics effect exists depends on a prediction of the standard model result to a comparable level

which in turns depends on the accuracy of the hadronic correction. A significant new physics signal, i.e., at the level of 2.4σ results if one adopts the estimates of Hagiwara *et. al.* for the hadronic correction. If one identifies supersymmetry as the origin of this deviation, then one finds that it leads to upper limits on sparticle masses within reach of the LHC for the $\mu > 0$ case. A positive μ is also favored by the $Br(b \rightarrow s + \gamma)$ constraint in the sense that the limits allow a large region of parameter space with light sparticle masses which are just what one needs to generate a large supersymmetric correction to a_μ . One drawback of a positive μ is that it is difficult to achieve Yukawa unifications for this case at least if one assumes universality of gaugino masses at the grand unification scale. This motivates us to explore non-universal gaugino mass scenarios where Yukawa unifications may be obtained for $\mu > 0$ along with a satisfaction of the muon $g - 2$ and the $Br(b \rightarrow s + \gamma)$ constraints. The sparticle spectrum in such scenarios again turns out to be low lying and can be probed by the LHC. Further, current and future dark matter detectors can also probe fully the dark matter predicted by this scenario in the direct detection dark matter experiments.

SUGRA and string models under the constraint of radiative electroweak symmetry breaking have two important branches: the ellipsoidal branch and the hyperboloidal branch. The former is valid for small $\tan\beta$ (< 5), whereas the latter exists for larger $\tan\beta$. Again the hyperbolic branch has two distinct sectors: the focus point (FP) region and the inversion region (IR). The focus point region corresponds to relatively low values of m_0 and $m_{1/2}$. Here, the lightest neutralino has a significant amount of the Higgsino component. On the other hand, the inversion region of the hyperbolic branch has relatively larger m_0 and $m_{1/2}$ while μ is still small. In this region the lightest neutralino is almost purely higgsino and most of the sparticles are heavy with masses in the several TeV region except for $\tilde{\chi}_1^0$, $\tilde{\chi}_2^0$ and $\tilde{\chi}_1^\pm$. Quite remarkably on the hyperbolic branch the relic density constraint from WMAP are still satisfied for both the focus point and the inversion region. Further, even though a large part of this region may lie outside the reach of the LHC a significant part would still be accessible to dark matter experiments of direct detection type.

Finally we have also discussed in this review the low energy signatures of the modular invariant soft breaking within the heterotic string frameworks while using the radiative electroweak symmetry breaking constraint. The use of the latter constrains the models significantly by fixing $\tan\beta$. Remarkably the neutralino relic density satisfies the WMAP cold dark matter

limits in a significant region of the parameter space of such models. Further the analysis leads to upper limits of sparticle masses for $\mu > 0$ under the combined constraints of WMAP and $Br(b \rightarrow s + \gamma)$. It is observed that the $\mu > 0$ case leads to the allowed parameter space being dilaton dominated and almost all the sparticles are found to lie within the reach of the LHC. Further, it is seen that the direct detection experiments via spin independent scattering ($\sigma_{\tilde{\chi}_1^0 p}^{SI}$) will completely probe such models.

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